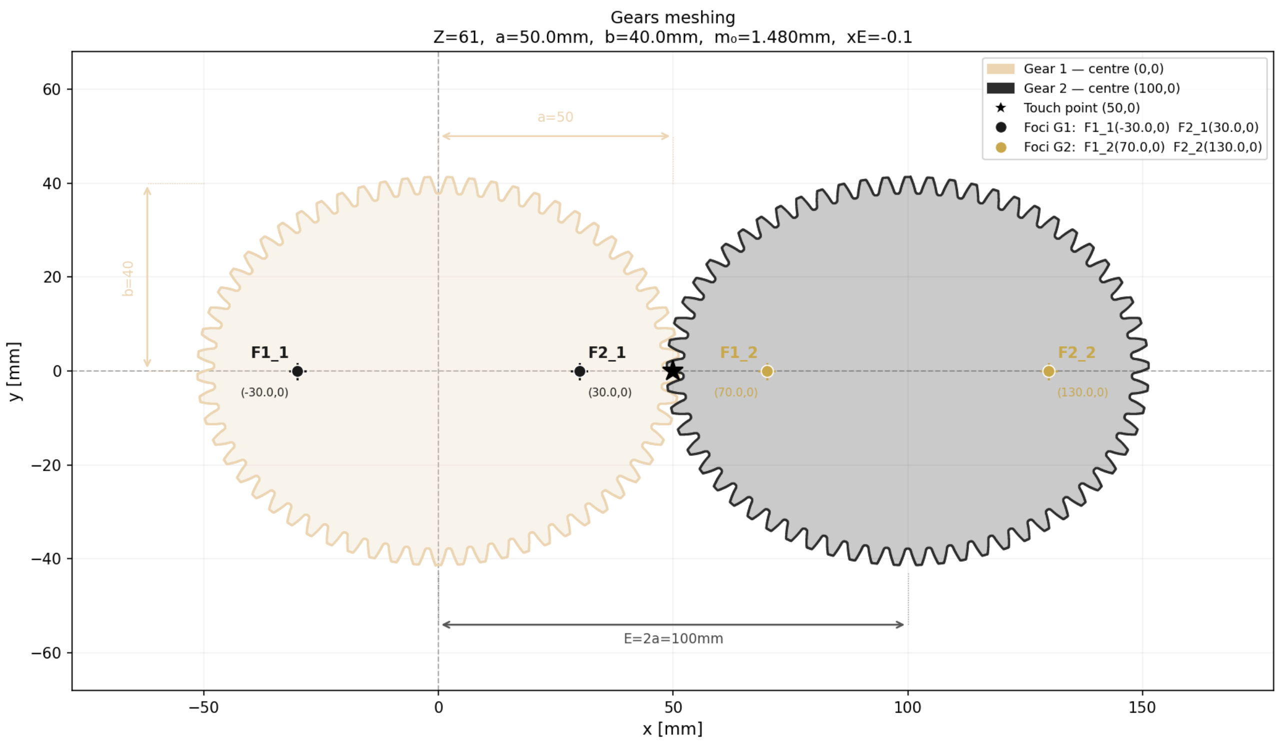
Elliptical and oval gears



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1.2 Abbreviations and symbols

Abbreviation, symbol	Explanation
a	Semi-major axis of the ellipse
b	Semi-minor axis of the ellipse
c	Distance from ellipse centre to focal point
e	Eccentricity, e = c / a
E	Centre distance between the two gears, E = r1 + r2 = constant
i	Momentary (instantaneous) transmission ratio
i(ϕ)	Instantaneous ratio as a function of polar angle phi
i_max	Maximum transmission ratio
i_min	Minimum transmission ratio
L	Circumference of the centrode
m	Module
N	Number of lobes (oval gear)
O	Geometric centre of the gear
O1, O2	Focal points; rotation centres of the elliptical gears
p	Semi-latus rectum, p = a * (1 - e^2)
r(phi)	Instantaneous pitch radius as a function of polar angle phi
r1(phi)	Instantaneous radius of gear 1
r2(phi)	Instantaneous radius of gear 2
R_max	Maximum radius of the oval gear centrode
R_min	Minimum radius of the oval gear centrode

z	Number of teeth
phi	Polar angle / rotational angle of the driving gear
phi1, phi2	Rotational angles of gear 1 and gear 2 respectively
omega1, omega2	Angular velocities of gear 1 and gear 2

1.3 References

[1] Aaron Fagan, Noncircular Gears, The Unicorn of Machine Technology, Gear Technology, June 2022.

[2] Litvin et al., Noncircular Gears, Design and Generation, Cambridge University Press, 2009

2 Noncircular gears

Noncircular gears are found as early as the sketches of Leonardo da Vinci in his Codice Atlantico, documenting his work from 1478 to 1519. The gear community is of course familiar with this, refer to e.g. Aaron Fagan' Addendum column "Noncircular Gears, the unicorn of machine technology", [1]. Their use is limited; they are niche component due to their complexity of design and manufacturing. The advent of servo motors solved the problem of controlled motion in a more general approach, reducing the need for noncircular gears further. Still, they remain high-performing mechanical solutions. The dedicated book "Noncircular Gears, Design and Generation" is widely known and gives a dense overview on the topic, [2]. In the context of this paper, we will limit the observations to convex elliptic, Figure 2 and oval gears Figure 3. Gear 1 and gear 2 have same properties.

These two types of noncircular gears transform motion between parallel axes where a continuously changing transmission ratio results within a single revolution. Unlike linkages mechanisms but similar to cam-follower mechanisms, elliptical and oval gears provide a compact arrangement for delivering periodic speed variations, making them suitable in e.g. packaging systems, textile machines, flow meters or heavy press machines, with slow working stroke and a fast return stroke. This document focuses on the differences between elliptical gears and their oval cousins. In extremis, both are transformed to circular gears, Figure 1.

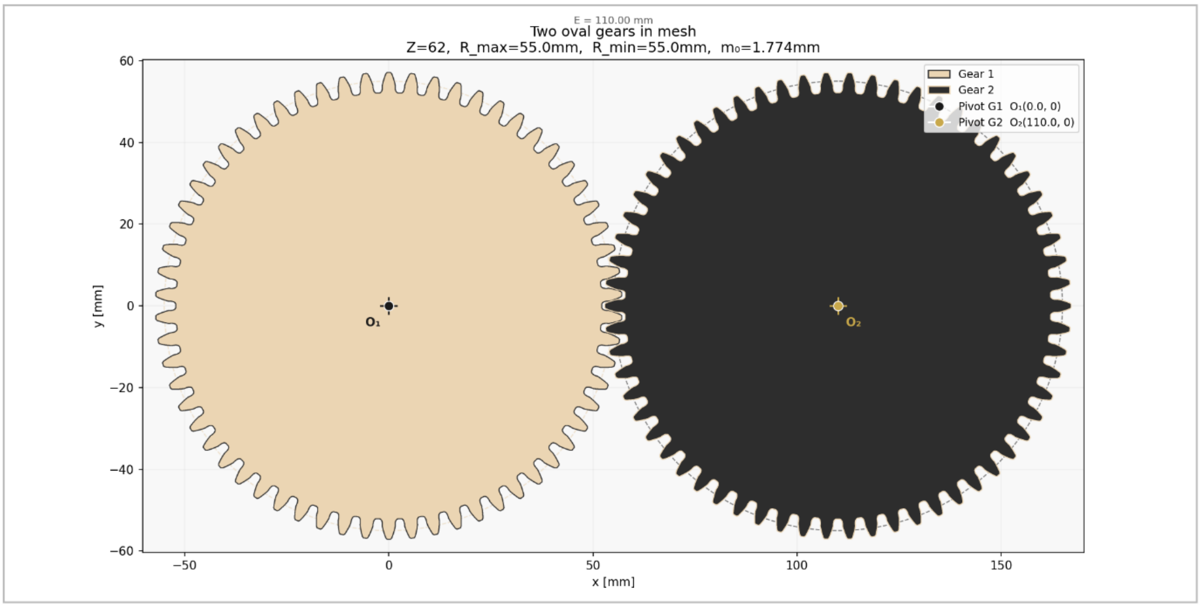

Figure 1 Two circular gears in mesh. Gear 1 is the driver, the gear on the left, rotating at constant speed. Gear 2, driven, is rotating at constant speed (neglecting manufacturing errors, teeth deformation and so on).

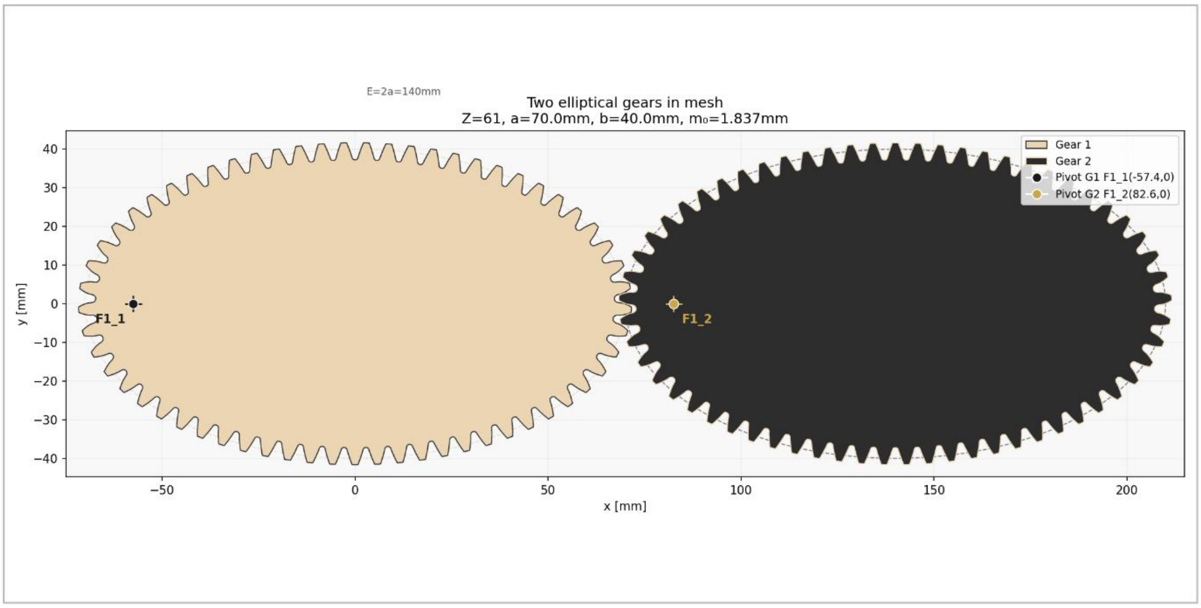

Figure 2 Two ellipticals gears in mesh. Again, gear 2 will not rotate at constant speed. Ratio in this configuration changes from i_{max}=10.2 to i_{min}=0.1. Note that both gears start the meshing cycle with major axes aligned.

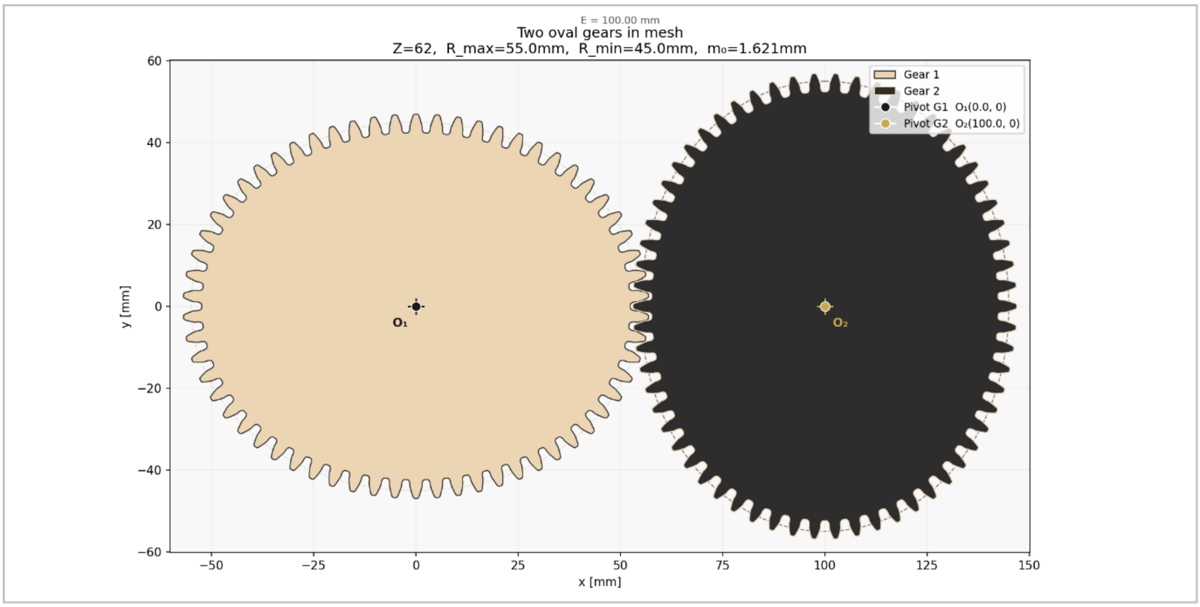

Figure 3 Two oval gears in mesh. Obviously, gear 2 will not rotate at constant speed. Ratio in this configuration changes from i_{max}=2.0 to i_{min}=0.5. Note that both gears start the meshing cycle with major axes in a rectangular arrangement.

The defining feature of non-circular transmissions is the momentary (instantaneous) ratio i. In circular gears, the ratio i is constant and may be determined from the number of teeth, Figure 4. In elliptical, Figure 5, and oval gears, Figure 6, it is a continuous function of the polar angle of rotation ϕ, $i(\phi) = \omega 2 / \omega 1 = r1(\phi) / r2(\psi)$. With the sum $E = r1 + r2$ = constant, the ratio is a function of the driver's radius alone $i(\phi) = r1(\phi)/(E - r1(\phi))$.

2.1 Circular gears rotate around their centers

Obviously, the ratio of two circular gears is constant and while trivial, the momentary ratio is shown in below figure for reference.

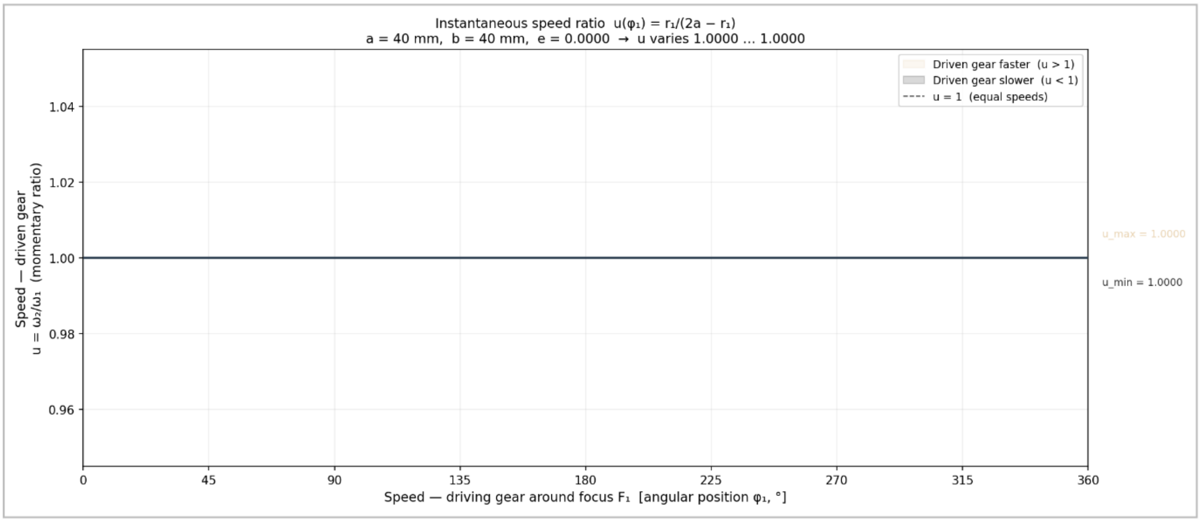

Figure 4 Momentary ratio expresses as speed ratio for the centrodes rotating without slip. For circular centrodes.

2.2 Elliptical gears rotate around the focal point

The elliptical gear is defined by a pitch curve (centrode) as a standard ellipse. A significant kinematic difference exists to circular gears rotating around an eccentric axis: to maintain a constant center distance E while rolling without slip, a pair of identical elliptical gears must rotate about one of their foci O_1, O_2 rather than their geometric centers O as with circular gears.

This requirement stems from a geometric property of the ellipse that for any point on the pitch curve, the sum of the distances to the two foci is constant and equal to the major axis length 2 * a. When the rotation centers are placed at the foci, the sum of the instantaneous radii of the two ellipses $r_1 + r_2$ equals the axial distance $E = r_1(\phi) + r_2(\psi)$, ϕ and ψ being the rotational angles of the two ellipses. For identical elliptical gears where one revolution of the driver corresponds to one revolution of the driven gear, the center distance is E = 2 * a, where a is the major axis length.

In polar coordinates, the elliptical centrode rotating about its focus is $r(\theta) = p / (1 - e * \cos(\theta))$ with $p = a * (1 - e^2)$, a is the semi-major axis, and e = c / a, the eccentricity and c is the distance from ellipse center to focal point. This rotation about the focal point results in a single, smooth speed fluctuation cycle per revolution, Figure 5.

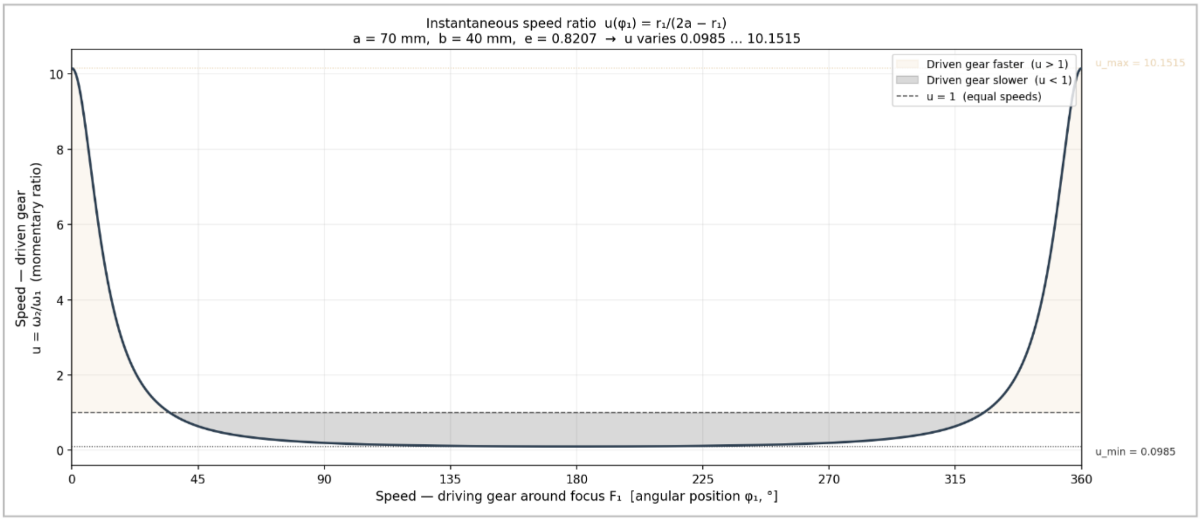

Figure 5 Momentary ratio expresses as speed ratio for the centrodes rotating without slip. For elliptical centrodes.

With the eccentricity chosen, the ratio spread is 10.1515 / 0.0985 = 103. The dynamics in a real-world application will be interesting…

The momentary ratio course is a function of a / b (major to minor half axis), as shown below for one of the horizontal axes, Figure 6. For a / b = 1, we have a circle, the ratio is then constant at I = 1.00, see left front edge of the color plot. In dashed red, the current design is shown. As the ellipse gets slimmer and slimmer (a / b increasing), the ratio spread goes towards several orders of magnitude (note that the vertical axis is logarithmic).

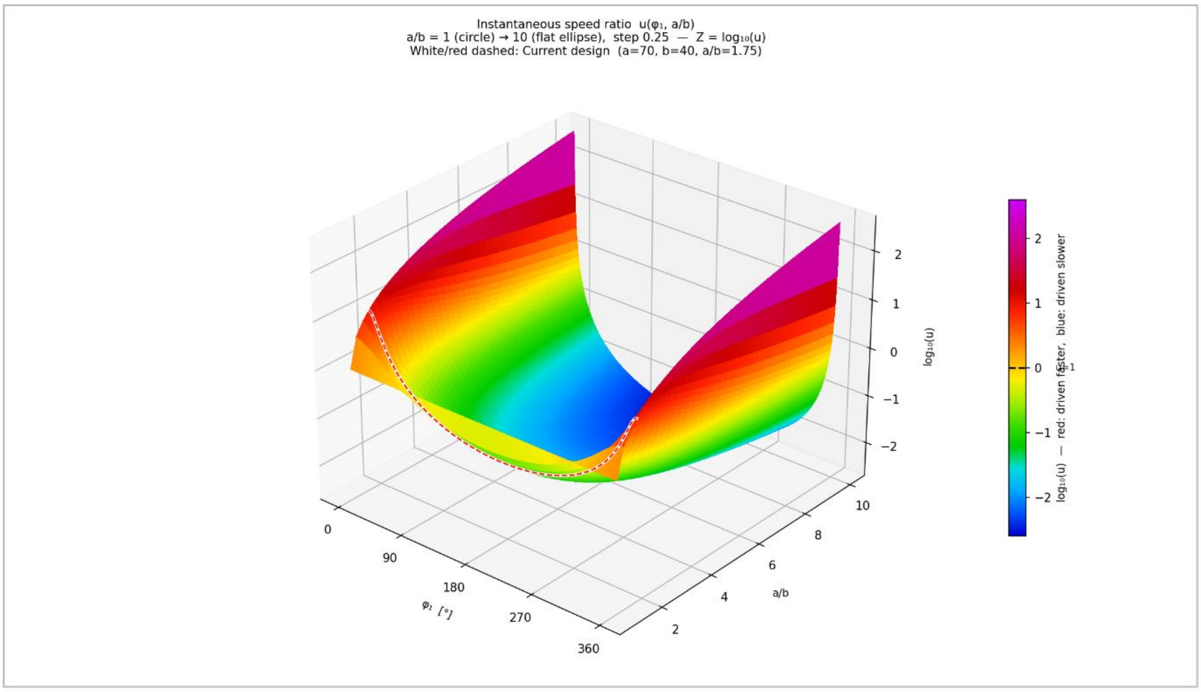

Figure 6 Ratio between two elliptic gears as a function of the rotational angle of the driving gear and the ratio a / b.

2.3 Oval Gears rotate around their center

While a elliptical gears rotating around the focal points result in one speed cycle per revolution, industrial applications may require multiple speed cycles. This requirement led to the development of the oval gear, which is technically a modified elliptical gear characterized by multiple lobes (in our case just two).

Oval gears rotate about their geometric centers O. The polar equation for the oval gear centrode is $r_1(\phi) = p / (1 + e * \cos (2 * N))$. A two-lobed oval gear ($N = 2$) will produce two ratio cycles per revolution of the driving gear, Figure 7. This symmetry ensures that the gears are balanced, a critical factor for the high-speed rotation.

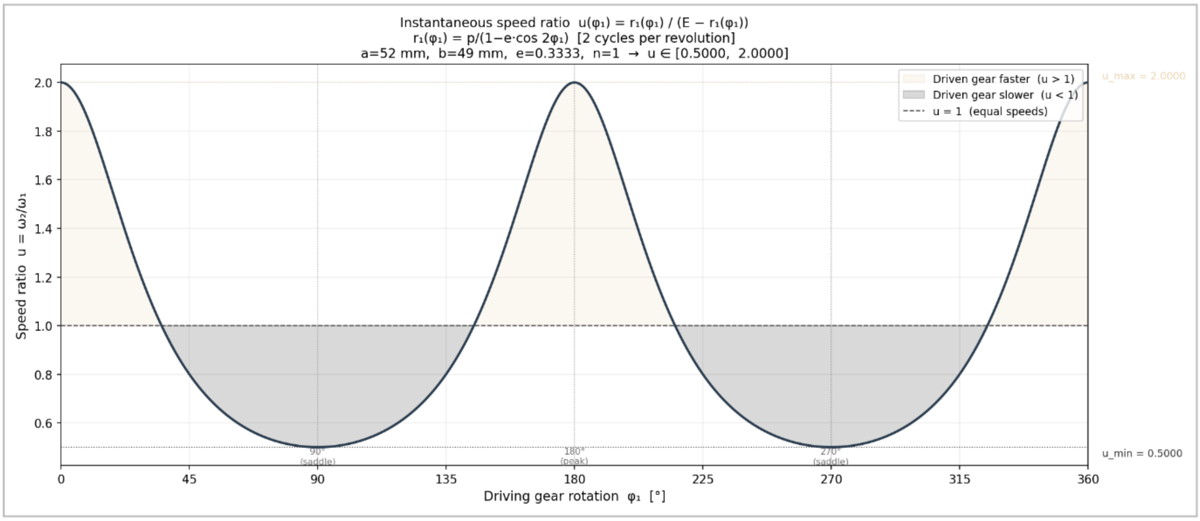

Figure 7 Momentary ratio expresses as speed ratio for the centrodes rotating without slip. For oval centrodes.

For oval gears, if the ratio between maximum radius R_{max} and minimum radius R_{min} is too high, the centrode turns partially concave, Figure 8. Concave centrodes are more difficult to manufacture; they require a shaping cutter type tool (or e.g. wire erosion) as opposed to a rack type tool. The centrode remains convex if the oval gear has radii that fulfill the condition $R_{min} > R_{max} * (1-2 / N^2)$, or, with $N = 2$, $R_{max} / R_{min} < 2.0$. In Figure 10, the geometry for $R_{max} / R_{min} = 2.0$ is shown. If the ratio exceeds this limit, the centrode has concave areas, Figure 8.

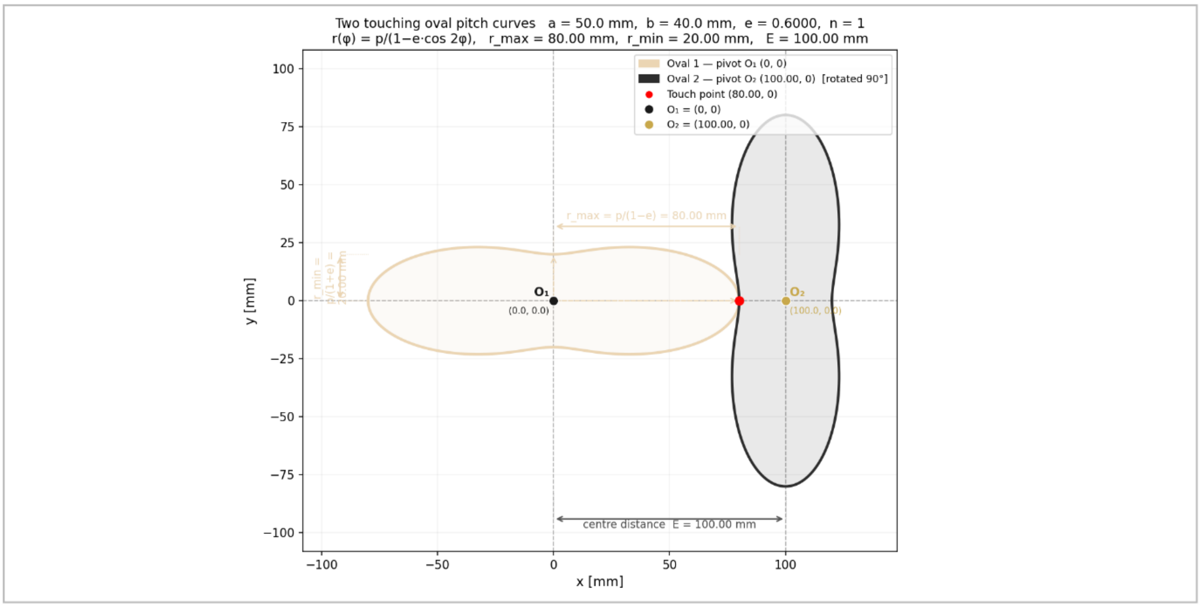

Figure 8 Two oval gears in mesh. $R_{max} > 2 * R_{min}$, centrode develops a slim waist.

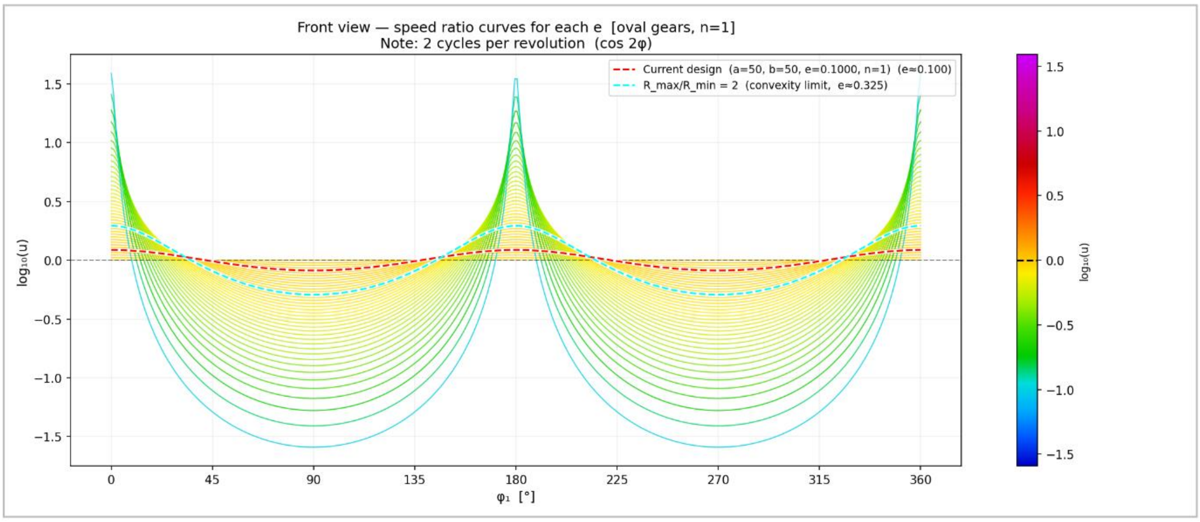

Figure 9 Momentary ratio for different oval gears (blue dashed line = current design, red dashed line = $R_{max} = 2 * R_{min}$).

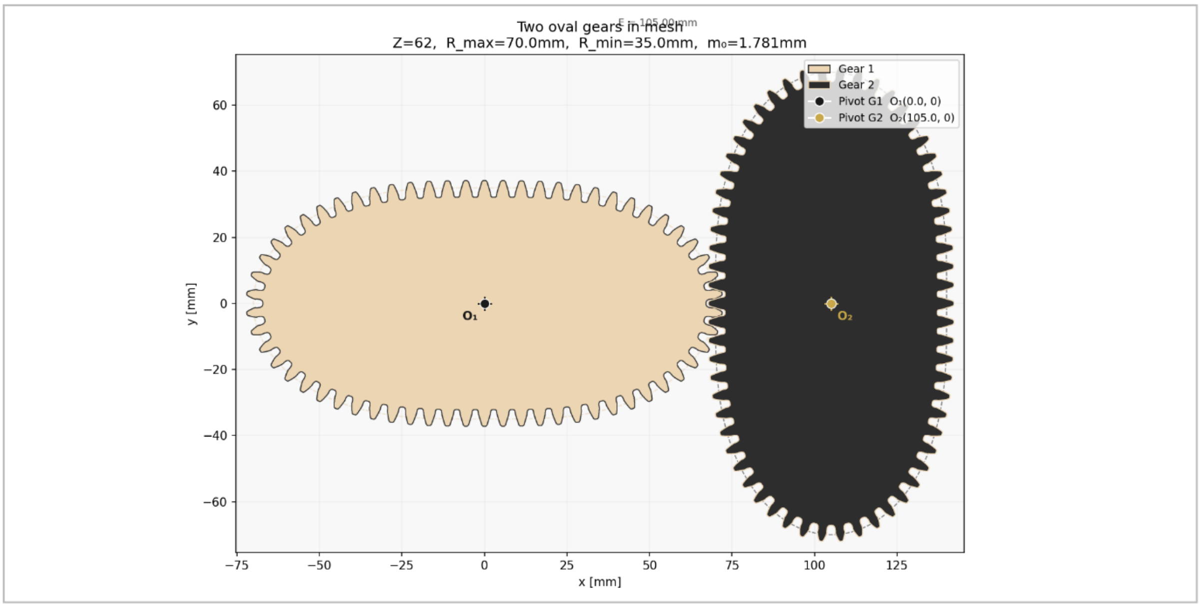

*Figure 10 Oval gears with R_{max} = 2 * R_{min}. The slimmest shape bevor concave shape occurs.*

3 Gear generation with a rack type tools

3.1 Basic rack definition

Tooth geometry is generated with a rack type topping tool, Figure 11. The cutter (manufacturing profile shift is applied for backlash) has a straight reference line that rolls on the centrode without slip. The contact point is the momentary center of rotation for the rack. The movement of the rack equals the arc length of the centrode when the contact point travels from the start position to the current position.

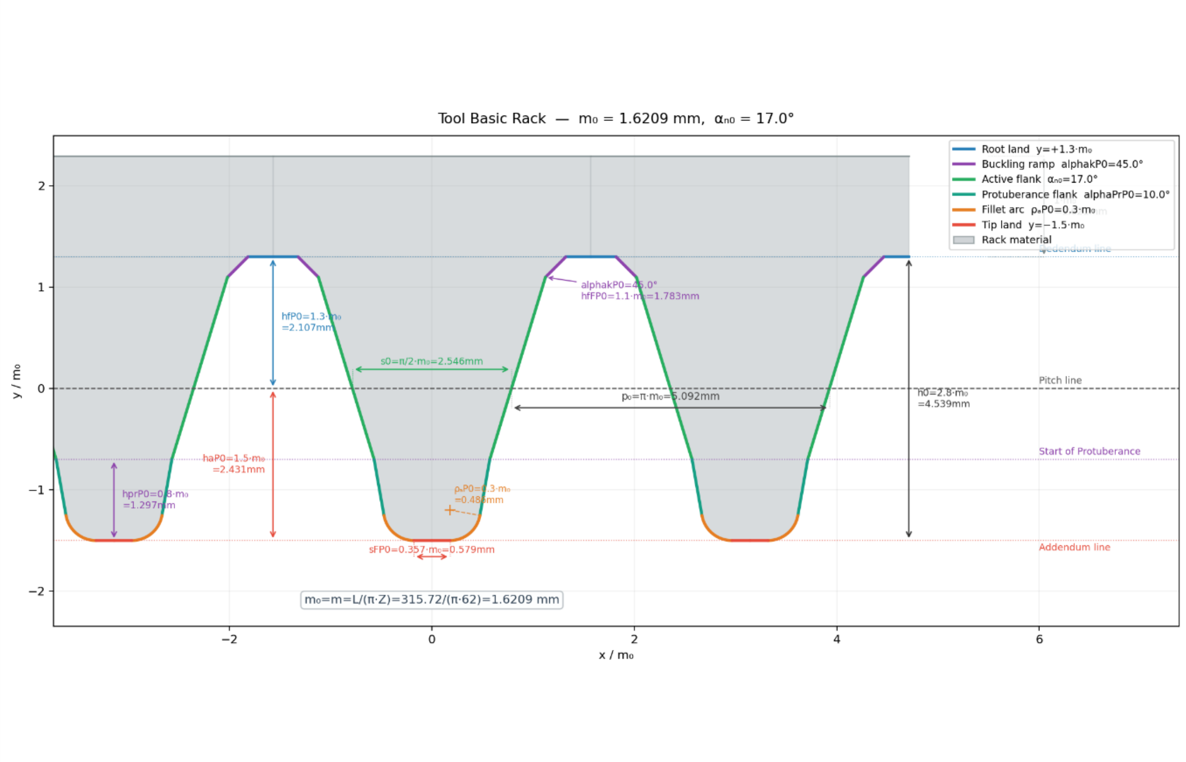

Figure 11 Rack definition.

3.2 Number of teeth

For non-circular gears to operate continuously, the circumference L of the centrode must be exactly equal to an integer number of teeth z times the pitch, L = m * z, where m is the module chosen to satisfy the condition.

Elliptical gears, require an odd number of teeth. This ensures that if a tooth is centered on say the left major half axis a gap is present on the right major half axis. Into this gap, the tooth on the left major half axis of the mating gear will fit. Note that major axes are aligned at start.

For oval gears, the number of teeth must be even but not divisible by four. This ensures there are two teeth on the major axes and two gaps on the minor axes, again fulfilling the meshing condition as at the start of mesh, the gear major axes are arranged perpendicularly.

3.3 Calculation procedure

Calculations were implemented using Python in Google Colab environment.